

Problem Sheet 2 For Supervision in Week 9

1. Find the quotient q and remainder r on dividing the following numbers by 17.

(i) 100, (ii) 1001, (iii) -44 , (iv) -99 .

2. ★ Find the great common divisors of the following pairs

(i) $(97, 157)$, (ii) $(527, 697)$,
(iii) $(2323, 1679)$, (iv) $(2821, 4247)$.

In each case write the greatest common divisor as a linear combination of the two initial numbers.

3. Find the greatest common divisors of

(i) 15691 and 44517,
(ii) 173417 and 159953.

If you have plenty of time to spare write the greatest common divisor as a linear combination of the two initial numbers.

For the following two questions recall that two integers a and b are coprime if, and only if, $\gcd(a, b) = 1$. This happens if, and only if, there exist integers m and n such that $ma + nb = 1$.

4. Find m and n to show that

(i) 41 and 68 are coprime, (ii) 71 and 118 are coprime.

Prove that $3k + 2$ and $5k + 3$ are coprime for all $k \in \mathbb{Z}$

5. Prove that if $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$ then $\gcd(ab, c) = 1$.

6. Find **an** integer pair of solutions (m, n) to each of the following:

(i)★ $3m + 5n = 1$,

(ii) $2m + 15n = 4$,

(iii)★ $31m + 385n = 1$,

(iv) $41m + 73n = 20$.

(v)★ $93m + 81n = 3$,

(vi) $697m + 527n = 13$,

(vii)★ $533m + 403n = 52$.